O'Halloran, K. L. (2010). The Semantic Hyperspace: Accumulating Mathematical Knowledge across Semiotic Resources and Modalities. In Christie, F. & Maton, K. (eds). Disciplinarity: Functional Linguistic and Sociological Perspectives. London: Continuum, 217-236.

Chapter 11

The Semantic Hyperspace: Accumulating Mathematical Knowledge across Semiotic Resources and Modalities

Kay L. O'Halloran

Introduction

The nature of mathematical language and the linguistic challenges of learning mathematics have been explored since the 1970s. Schleppegrell (2007) provides a comprehensive synthesis of the research completed by applied linguistics and mathematics educators which aims to identify the linguistic structures in mathematics and how these differ from everyday language. In particular, Michael Halliday (1978) developed the notion of the mathematical register to explain how language is used to construct mathematical knowledge in ways that differ from other academic subjects. Halliday (1978: 195-196) explains that mathematics is not just a question of new words, but it is also new 'styles of meaning and modes of argument ... and of combining existing elements into new combinations'. From here, systemic linguists investigated the grammatical forms of the scientific register (e.g. Halliday, 2006; Halliday & Martin, 1993; Martin & Veel, 1998; Veel, 1999) using Halliday's (2004) systemic functional grammar. More recently, researchers working in the systemic functional tradition have expanded the focus on language to view mathematics as a multisemiotic discourse constructed through language, image (e.g. diagrams and graphs) and mathematical symbolism (e.g. Lemke, 2003; O'Halloran, 1999, 2005, 2009). Current trends indicate the systemic approach, located within the wider domain of multimodal analysis and multimodality (e.g. Bednarek & Martin, 2010; Jewitt, 2009), holds much promise for future research in mathematics education (Clarke, 2001; Schleppegrell, 2007).

The aim of the present chapter is to contribute to the discussion of the multisemiotic nature of mathematics by investigating how mathematical knowledge is accumulated across semiotic resources (language, images and mathematical symbolism) and modalities (oral, visual, haptic and others) in the classroom. The focus is the accumulation of mathematical knowledge as mathematical texts are constructed, taking into account the mathematical discourse itself, the written texts which students read, learn and reproduce at school, and the spoken discourse which takes place as these texts are constructed in classroom activities.

The specific areas explored in this chapter are:

- *intra-semiosis* and the functionalities of language, mathematical images, and mathematical symbolism;
- *inter-semiosis* and the meaning arising from the integration of language, images and symbolism in mathematics texts;
- *inter-modality* and the overlapping modalities (visual, aural, haptic and others) through which semiosis takes place; and
- the *semantic hyperspace* arising from the integration of semiotic resources across modalities.

This investigation is undertaken with the aim of deepening our understanding of mathematical knowledge and how it differs from other forms of knowledge. The findings have implications for mathematics teaching and learning.

Mathematics and Semiotic Resources

(a) Intra-Semiosis: Language, Image and Mathematical Symbolism

Mathematics texts are constructed using language, mathematical images (e.g. graphs, geometrical diagrams and forms of visual representation) and mathematical symbolism. Following Halliday (1978, 2004), these three building blocks of mathematics knowledge are called semiotic resources, which are the system of signs used to create meaning in mathematics. Halliday (2004) explains that semiotic resources have the potential to realise four types of meaning which he calls metafunctions: experiential meaning (to construct our experience of the world), logical meaning (to construct logical relations in that world), interpersonal meaning (to act on others) and textual meaning (to organise the message to achieve the desired functions). Semiotic resources must have an underlying organisation in order to realise the four metafunctions, and this organisation is the set of inter-related systems called the systemic grammar (Halliday, 2004). The four metafunctions are realised through choices from systemic grammar of each semiotic resource.

Intra-semiosis refers to the unique functionality of each semiotic resource arising from its systemic grammar. The intra-semiotic semantic potential of language, mathematical symbolism and images and their grammatical systems have been discussed in detail elsewhere (Lemke, 2003; O'Halloran, 2005, 2007). Therefore, only a summary of the functions of the three resources and their grammatical systems is provided below.

Mathematical language has many roles in mathematics, but generally it plays an important contextualising function with regards to introducing and explaining mathematical knowledge. Typical uses include the explanation of new theory and concepts, introduction of problems, instructions regarding the procedures involved in solving the problems and the discussion of results. Typically, experiential and logical meaning are expanded through careful textual organisation, accompanied by regular patterns of interpersonal meaning with a high truth-value. The interpersonal orientation and the precise textual organisation permit the focus of semantic expansion to be the experiential and logical content of the mathematics text.

Halliday (1993) explains that the difficulties in understanding the experiential and logical content of scientific language include interlocking definitions, technical taxonomies, special expressions, lexical density, syntactic ambiguity, semantic discontinuity and grammatical metaphor. In particular, grammatical metaphor has led to a semantic shift in scientific writing whereby simple linguistic constructions found in everyday language (e.g. *X is related to Y so Z occurs*) are encoded as metaphorical participants and relational processes (*the relationship X and Y results in Z*). The semantic shift may result in two or more clauses being encoded in a single nominal group (*the result Z of the relationship X and Y*). The condensation of experiential and logical meaning into relational processes and nominal groups aids the rhetorical organisation of the text and the development of scientific argumentation (Halliday, 2006; Halliday & Martin, 1993).

Mathematical symbolism, on the other hand, is the semiotic resource through which mathematical problems are solved. Having derived from natural language and material actions (e.g. measuring, counting and dividing), modern mathematical symbolism retains some linguistic selections. However, the grammar of the mathematical symbolism is organised in such a way that the relations between the mathematical participants can be rearranged and simplified so that the symbolism becomes a specialised tool for logical reasoning. The grammatical strategy found in mathematical symbolism is the opposite of that found in scientific language. Rather than encoding meaning into long nominal groups like language, the grammar of mathematical symbolism works through deep embedding of configurations of mathematical participants and processes. The preservation of mathematical participants and the processes means that they can be reconfigured to solve problems, according to pre-established results, laws and axioms. The mathematical symbolism has a range of grammatical strategies which makes the preservation, rearrangement and simplification of mathematical processes and participant configurations possible, such as generalised participants, use of spatial notation (for example, division and powers) and brackets, ellipsis of processes and rules of order which stipulate the sequence in which the mathematical processes unfold. The sequence of unfolding processes in mathematical statements is not linear, but it is predetermined in specific ways by mathematical rules. Indeed mathematical symbolism is a tool carefully designed to aid logical reasoning through the precise encoding of mathematical participants and processes in a format which facilitates their rearrangement.

Mathematical images (graphs, geometric diagrams) provide a semantic link between the linguistic description of the problem and the symbolic solution. That is, mathematical images provide an intuitive overview of the relations between mathematical participants which are viewed as parts of the whole. From here, the symbolism is used to derive the solution to problem using its specialised grammar and mathematical laws, axioms and pre-established results. As a consequence of this semantic circuit between language, image and the symbolism, mathematical graphs and diagrams contain linguistic and symbolic elements. In fact, most parts of mathematical texts contain various combinations of linguistic, symbolic and visual components, which illustrates that the inter-semiotic relations between the three semiotic resources is an important foundation for the construction of mathematical knowledge.

(b) Inter-semiosis: Language, Image and Symbolism

The semantic circuit between language, image and mathematical symbolism involves discourse patterns where the resources are called upon to fulfil particular functions. For example, there may be a shift from language (to introduce theoretical concepts or a problem), to image (to view the relations between the mathematical participants) to mathematical symbolism (to capture the relations between the participants and to solve the problem). Language and images are typically used to introduce and conceptualise mathematical concepts and problems, and the symbolism comes into play to formalise those relations and solve the problem. More recently, the semantic circuit in mathematics has changed with increasing power of computers and the development of scientific visualisation techniques (O'Halloran, 2009). For example, the computational results of symbolic algorithms are displayed as complex interactive visualisations. The semantic circuit between language, image and symbolism still exists, but in a form that is different from that developed in modern written mathematics.

The inter-semiotic integration of the resources in mathematics texts is a site for immense semantic expansion because resemioticisation (Iedema, 2001) from language to image to symbolism means that new sets of grammatical systems with their own meaning potential to come into play as the discourse moves from one semiotic resource to another. In addition, the potential exists for a semantic shift in functional elements (e.g. a linguistic process becomes a visual participant) and the introduction of new elements which previously did not exist. Therefore, the systemic grammars of language, image and symbolism, each with their own semantic potential, need to be understood in relation to the inter-semiotic semantic circuit which expands the semantics of the mathematics beyond the sum of the meaning potentials of the three resources. The intra-semiotic potential of each resource and inter-semiotic expansions of meaning are so powerful that mathematics has been used to rewrite the physical world. As mathematician Davis (2006) explains, mathematics is the science and art of quantity, space, and pattern. Its materials are organized into logically deductive and often computational structures where ideas are abstracted, generalized, and applied to concerns other than mathematics itself. The semantic expansions made possible through mathematics have led to life as we know it today.

It is important to recognise that mathematics developed as a visual form of representation (written and printed) so that new grammatical systems which exploit spatiality as a resource for encoding meaning have been developed for mathematical images and the symbolism (e.g. division realised as one participant spatially positioned above another, index notation to represent how many times the base is multiplied by itself). These grammatical systems which use space to encode meaning extend beyond those found in language which has to function in both spoken and written contexts. In order to explore what happens in the classroom when spoken language is used to discuss the written mathematical text with its unique spatial grammar, it is necessary to differentiate the ways in which semiotic resources materialise. For this purpose, the concept of semiotic modalities is introduced.

Mathematics, Modalities and Inter-Modal Relations

The terms semiotic mode and modality are used in various ways in multimodal research, most typically in a manner which is interchangeable with the term semiotic resource (e.g. Baldry & Thibault, 2006; Bateman, 2008; Jewitt, 2009; Kress & van Leeuwen, 1996, 2001; O'Halloran, 2004b; van Leeuwen, 2005). However, as Bateman (submitted for publication) points out, the term semiotic mode (and modality) are most often used loosely and in an ill-defined manner. To overcome this problem and move beyond pure description, Bateman provides a comprehensive model for semiotic mode, which relates the physical substrate of the sign with the abstract semantic plane. Bateman's (submitted for publication) model closely follows Michael Halliday's (2004) concept of semiotic resource, except that Bateman formally and semantically relates sign choices in terms of their inter-semiotic activity in multimodal texts.

In this paper, the terms mode and modality are used somewhat differently to distinguish between (a) semiotic resources and (b) the sensory modalities through which the semiotic resources materialise, following the use of the term modality in human-computer interaction studies. The major modalities are the visual modality (visual perception), the aural modality (hearing) and haptic modalities (tactile and proprioception which are the senses of touch and the perception of body awareness respectively). Other modalities include gustation (sense of taste), olfaction (sense of smell) and modalities involving perceptions of temperature, pain and the sense of balance. For the purposes of this chapter, modalities are grouped into four categories: visual, aural, haptic and others, with a focus on the visual and aural modalities in the classroom.

The distinction between semiotic resources and sensory modalities permits inter-semiotic activity to be correlated with inter-modal relations across the different modalities. This interplay between semiotic resources and modalities is significant in mathematics classrooms where learning mathematics involves moving from the concrete world of multiple resources and modalities (e.g. language, image, gesture, action and aural, visual and haptic modalities) into the abstract semantic space of mathematics with its three semiotic resources which function in the visual mode. Historically, the shift from the material to the abstract in mathematics took place as material activity (involving actions, gestures and speech) evolved into sign systems (e.g. tally marks and symbols) and counting technologies (e.g. beads, cones and abacus) which eventually led to modern mathematics. The distinction between semiotic resources and modalities permits the interaction of systems on the expression plane (for example, tone in spoken language) to be mapped onto grammatical choices (e.g. Mood system) giving rise to new systems (e.g. Halliday's key system). In much the same way, the potential to develop inter-semiotic and inter-modal systems which operate across semiotic resources and modalities is possible if semiotic resources are differentiated from modalities. As we shall see, inter-semiotic and inter-modal relations are significant because they result in metaphorical expressions of meaning in the classroom.

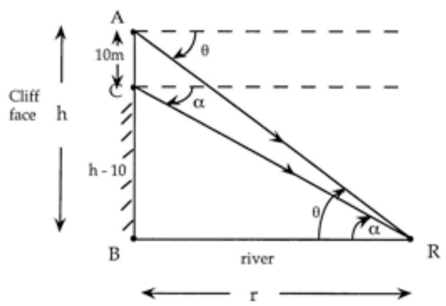
In what follows, the aural discourse (language) and the visual mathematical texts (language, image and symbolism) in a mathematics lesson are investigated in order to explore the nature of inter-semiotic and inter-modal expansions of meaning which take place in the classroom. Following this, the semantic spaces that subsequently arise in mathematics and the mathematics classroom are discussed.

Accumulating Mathematical Knowledge in the Classroom

The mathematics lesson under consideration is a trigonometry lesson that took place in a private boys school in Perth Western Australia. The lesson is described in detail elsewhere (O'Halloran, 1996, 2004a) so only brief description is given here.

The teacher introduces a trigonometric problem and proceeds to solve the problem on the board in a lesson which is part of the mathematics curriculum macrogenre (Christie, 1997, 2002) where students are expected to solve trigonometric problems in two and three dimensions using trigonometric ratios. The lesson may be classified theory/applications lesson because students are learning to apply the theory of trigonometric ratios to solve a problem. The problem in this case involves finding the algebraic expression for the height of a cliff and the width of a river, given two angle measurements. The segment of the lesson under consideration is board demonstration activity where the teacher works through the problem on the board by soliciting answers from the students. The teacher establishes congruent dominant tenor relations with the students, soliciting extended responses from them until the problem is solved. The lesson unfolds in a systematic manner, which is reflected in the board text displayed in Figure 11.1.

HOW HIGH and HOW WIDE ?



A man is at point A (on the cliff face) with a 10m rope and a device that measures angles.

How can the man determine the:

i. height of the cliff at A

ii. width of the river

Assume:

1. Cliff face is at right angles to the river.

2. Error of measuring angles is $\pm 0.5^0$

Method :

Take 2 successive angles of depression, from a known distance apart.

In right Δ CBR:

$$\tan \alpha = \frac{h - 10}{r} \qquad (1)$$

In right Δ ABR:

$$\tan \theta = \frac{h}{r} \qquad (2)$$

Algebraically obtain ONE variable
as the subject of EACH equation :

From (1) : $h - 10 = r(\tan \alpha)$

$\therefore h = 10 + r(\tan \alpha)$

(2) : $h = r(\tan\theta)$

Equate expressions for h :

$10 + r(\tan \alpha) = r(\tan\theta)$ $\qquad 10 + 3x = 2x$

$10 = r(\tan\theta) - r(\tan \alpha)$

$10 = r(\tan\theta - \tan \alpha)$ $\qquad 10 + 5x = 2x$

$\qquad 10 = -3x$

$$\therefore r = \frac{10}{(\tan\theta - \tan \alpha)}$$

Substitute r into eq^n (2) :

$$h = \frac{10(\tan\theta)}{(\tan\theta - \tan \alpha)}$$

Suppose $\theta = 64.5^0$

$\alpha = 63^0$

Estimate $r = 40 \text{ -> } 400$

$h >> r$ $\qquad h =$

$h = 400 \text{ -> } 500$

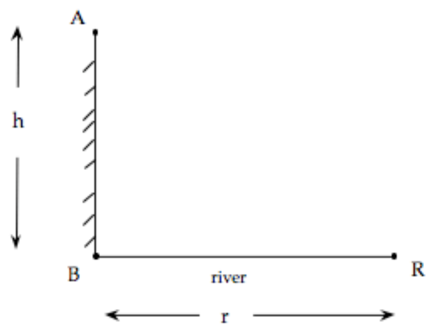
$r = 74.6$

$h = 166.44$

Figure 11.1 The Board Text

The spoken discourse was transcribed alongside the board text to capture inter-semiotic (language, image and symbolism) and inter-modal (aural, visual and haptic) relations as the mathematical problem is solved. In Figure 11.2, the diagram and board text appear on the left hand side, and the transcription with numbered clauses appears on the right hand side. The spoken discourse and the board texts were analysed using Halliday’s (2004) systemic functional grammar and systemic functional frameworks for mathematical images and the symbolism (O'Halloran, 1996, 1999). In what follows, extracts from lesson are used to investigate the semantic circuit and the inter-semiotic and inter-modal expansions of meaning which take place as the problem is solved. The relevant parts of the spoken discourse and the board texts are highlighted by rectangular overlays in Figures 11.1-7. For example, processes, participants and circumstance in the spoken discourse and the visual entity in the board text are highlighted through rectangular overlays in Figure 11.3.

HOW HIGH and HOW WIDE ?



A man is at point A (on the cliff face) with a 10m rope and a device that measures angles.

How can the man determine the:

i. height of the cliff at A
ii. width of the river

58	T	we'll note that down first
59	T	OK the two things [[that he has]]
60	T	he is currently [at position A]
61	T	so with this information [[he has a ten metre rope and a device [[that measures angles]]]] we are asking the question
62	T	how can the man determine firstly the height [of the cliff [at point A]] and secondly the width [of the river]
63	T	right so people there is our problem
64	T	so there are two things [[we have to find]]
65	T	and it's now a matter of /ahh thinking about
66	T	how we can solve this
67	T	and if there's a number of suggestions [[that /ah arise]]
68	T	we will /ah look at them [in turn]

Figure 11.2 The Board Text and Spoken Discourse

The first type of inter-semiotic semantic shift from spoken language to visual image occurs when the teacher constructs the sides of the triangle on the board in Figure 11.3. The teacher says ‘if you look straight out there’ (clause 161), ‘that would form a right angle’ (clause 162). The spoken discourse is a nuclear configuration of participant ‘you’, process ‘look’ and circumstance ‘straight out there’. This linguistic nuclear configuration is resemioticised as a horizontal line segment in the diagram, so there is an inter-semiotic semantic shift from nuclear configuration in spoken language to a visual entity in the diagram (i.e. linguistic process, participant and circumstance → visual participant). The semantic shift is indicated in Figure 11.3 by the arrow linking the set of three boxes to a single patterned box marked ‘V’.

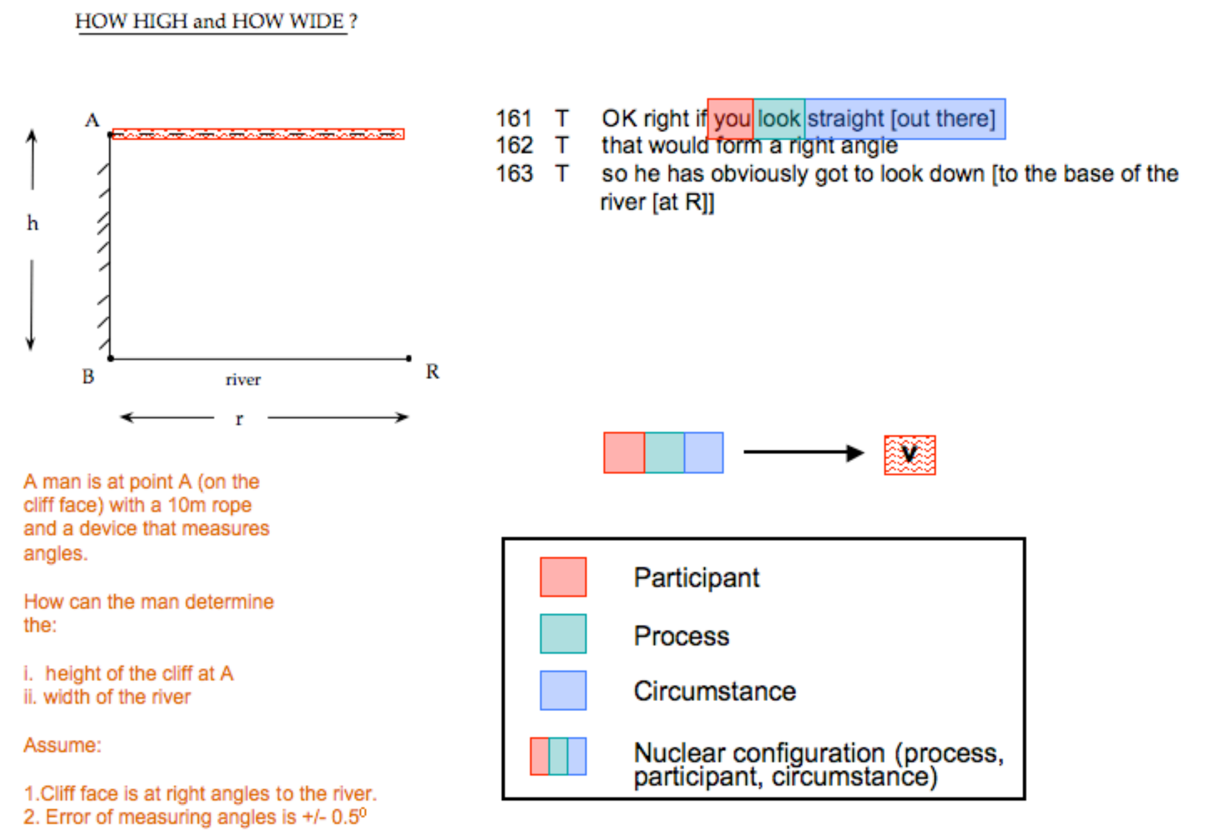


Figure 11.3 Inter-Semiotic Expansions between Spoken Language and Image (1)

A similar inter-semiotic semantic shift from language to image occurs in Figure 11.4 where the linguistic nuclear configuration ‘he has obviously got to look down to the base of the river at R’ (clause 163) is resemioticed as the line segment AR in the diagram. This inter-semiotic semantic shift from linguistic nuclear configuration to visual entity (i.e. linguistic process, participant and circumstance → visual participant) is indicated in Figure 11.4 by the second arrow which links the set of three boxes to a single box marked ‘V’. The resemioticisation of the spoken discourse in the classroom (language with aural modality) to the mathematical diagram (image with visual modality) represents a move from the material world of everyday activity into the abstract semiotic world of mathematical representation. Note that interpersonally, the linguistic statement in this case is highly modalised with obviousness through the mood adjunct ‘obviously’ and the finite element ‘got to’.

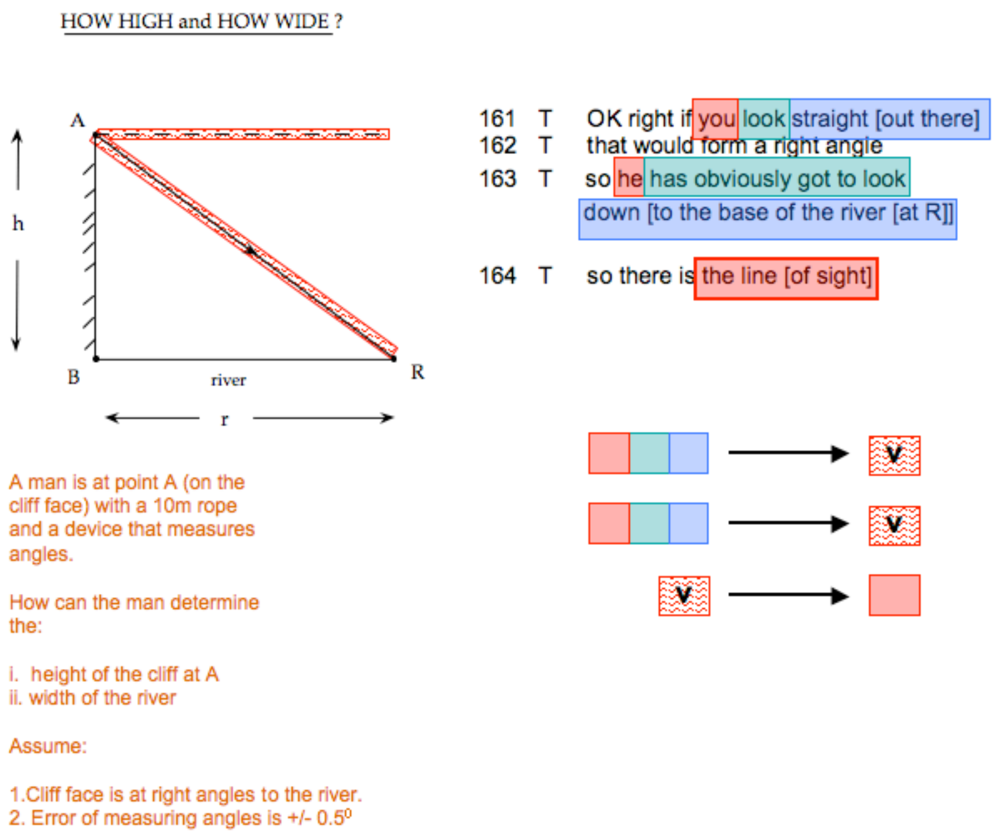
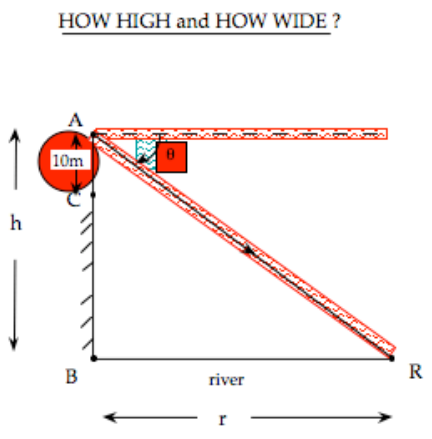


Figure 11.4 Inter-Semiotic Expansions between Spoken Language and Image (2)

Another form of inter-semiotic semantic shift takes place when visual elements in the diagram are resemioticised back into spoken language. For example, line segment AR in the diagram in Figures 11.3-4 originally obtained from the inter-semiotic shift from language to image (i.e. process, participant and circumstance → visual participant) is resemiotised back into spoken language when teacher refers to the line segment AR as ‘the line of sight’ (clause 164). The resemioticisation of AR as the linguistic entity ‘the line of sight’ is indicated in Figure 11.4 by the arrow linking the box marked ’V’ (visual participant) to the box (linguistic participant). The semantic circuit in this case (i.e. language to image to language) thus leads to nominalisation in language (i.e. process, participant and circumstance → visual participant → linguistic participant). Inter-semiotic shifts of the type have significance for our understanding of grammatical metaphor in language.

There is further evidence to suggest that grammatical metaphor in language arises from inter-semiotic and inter-modal activity. For example, the introduction of the horizontal line segment and the line of sight AR means that the triangle is constructed in the diagram. From here, two angles θ and α are obtained using a device that measures angles (see problem in Figure 11.1). The teacher states that 'we can measure that angle', resulting in the arrow (to represent the process of measuring) and the symbolic element θ (the size of the angle) in the diagram in Figure 11.5. The teacher states that 'using the rope as a way to be sure of some distance', the second angle α is obtained ten metres down the cliff face. The visual entity AC is thus introduced in the diagram (see Figure 11.5), and the distance is marked 10m. Therefore, the linguistic configuration 'using the rope as a way to be sure of some distance' (clause 171) leads to the introduction of visual entity AC (i.e. linguistic process, participant and circumstance → visual participant). The teacher then refers to this visual entity AC as a 'ten metre difference in height' (clause 179). The inter-semiotic and inter-modal semantic shift which takes place (i.e. linguistic process, participant and circumstance → visual participant → linguistic participant) also suggests that grammatical metaphors in language arise from inter-semiotic and inter-modal activity.

A further grammatical metaphor arising from inter-semiosis and inter-modality takes place. As discussed, the linguistic nuclear configuration 'we can measure that angle there' (clause 165) is resemioticed as an arrow in the diagram to realise a visual process (i.e. linguistic process, participant and circumstance → visual process) in Figure 11.5. The process of measuring results in the introduction of the symbolic entities θ and α (i.e. linguistic process, participant and circumstance → visual process → symbolic participant). The symbolic participants are subsequently resemiotised as the grammatical metaphor 'two angles of depression ten metres apart' (i.e. linguistic process, participant and circumstance → visual process → symbolic entity → linguistic participant). Perhaps historically, inter-semiosis and inter-modality have given rise to grammatical metaphors in scientific language.



A man is at point A (on the cliff face) with a 10m rope and a device that measures angles.

How can the man determine the:

i. height of the cliff at A
ii. width of the river

Assume:

1. Cliff face is at right angles to the river.
2. Error of measuring angles is +/- 0.5^0

168 T OK right and then you said ten metres below
169 A or five
170 T ten or five
171 T so <in other words> using the rope [as a way [[to be sure of some distance]]]
172 T let's assume
173 T he goes the full ten metres
174 A OK
175 T ahh and [on this diagram] the scale [for the cliff face] is usually / ahh quite large
176 T we'd assume
177 T and therefore let's say some other point here

178 T ahh let's call this point /ah C

179 T such that we have a ten metre difference [in height] here
180 T and I'll just [on the diagram] indicate ten metres [between points A and C]

Figure 11.5 Grammatical Metaphor as a Multimodal Phenomenon

The trigonometric problem is solved with the introduction of two angles θ and α because the tangent ratio is used to express the relationship between the size of the angles and the lengths of the sides of the triangle, thus permitting algebraic expressions for the height of the cliff and the width of the river to be found (see Figure 11.1). Previously established mathematical results (parallel lines axiom, properties of right triangles, definition of tangent and cotangent ratio and algebraic laws) are required to solve the problem, illustrating the vertical structure of mathematics knowledge. The larger discourse patterns arising from the semantic circuit in mathematics are illustrated in the move from linguistic and visual description of the trigonometric problem to its symbolic solution.

Further inter-semiotic expansions of meaning result from the move from the mathematical diagram to the symbolic solution. For example, the line segment CB (derived from the length of AC which is 10m) is resemioticised as (h - 10) in the symbolic equation for the tangent ratio tan α (see Figure 11.6). That is, the inter-semiotic shift from diagram to mathematical symbolism involves a resemioticisation of the visual entity BC into the symbolic nuclear configuration (h - 10) (i.e. visual participant → mathematical participants and process). This suggests that inter-semiotic shifts (in this case, image to symbolism) involve up-ranking semantic shifts where a lower rank (visual participant CB) is resemioticed at a higher rank (nuclear configuration of mathematical participants and process, in this case h - 10), a trend which is the opposite to the down-ranking semantic drift of grammatical metaphor in language. The downranking semantic drift in language may even be the result of the upranking semantic shifts which occur intersemiotically.

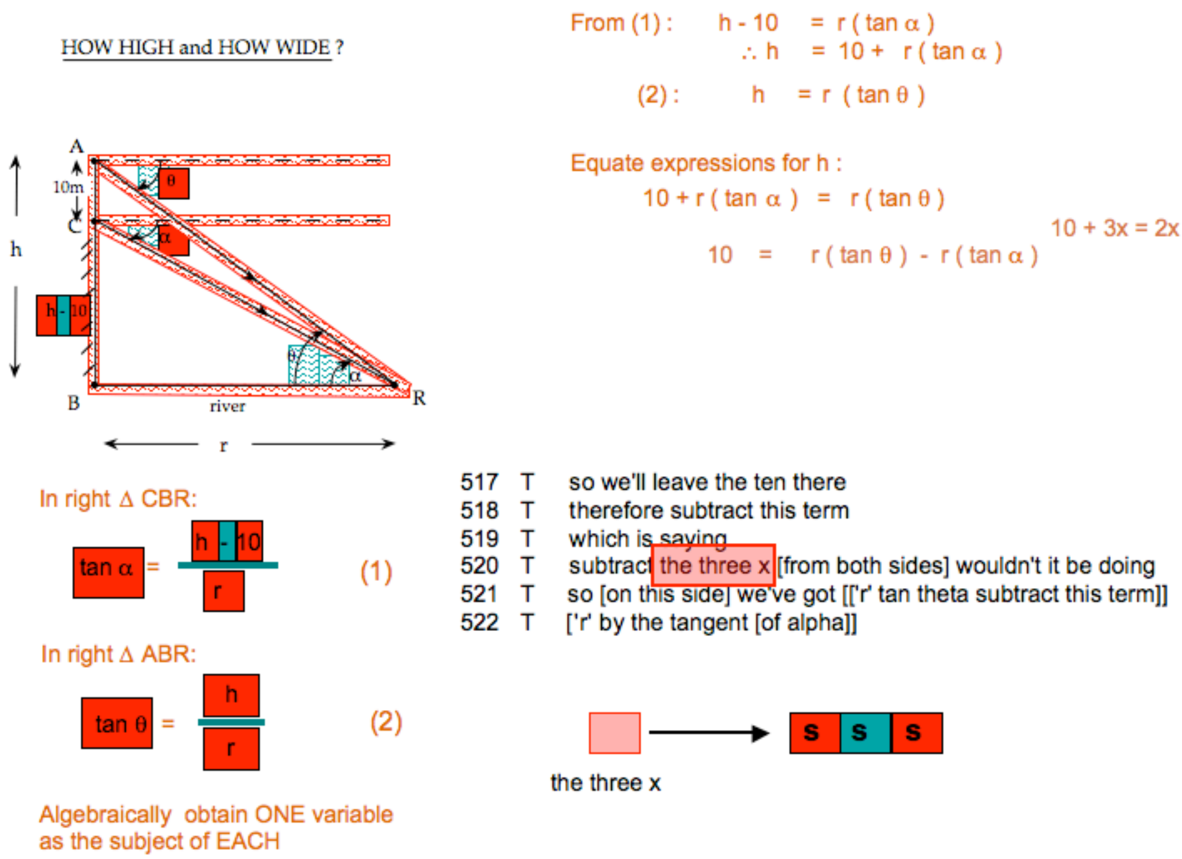


Figure 11.6 Inter-Semiotic Expansions between Image and Mathematical Symbolism

Finally, the verbalisation of mathematical symbolism in the spoken discourse in the classroom has the potential to lead to a semantic disjunction in that the spoken linguistic discourse means something different to written mathematical statement. For example, 'the three x' (clause 520) in Figure 11.7 is an entity (i.e. a nominal group). However, the symbolic $3x$ in Figure 11.1 is a nuclear configuration involving mathematical participants 3 and x and the operative process of multiplication (i.e. 3x is equivalent to 3 times x). Therefore there has been a semantic shift from the written '3x' to the spoken 'the three x' (i.e. mathematical process, participant → linguistic participant). The spoken version of the written mathematical symbolism has a different meaning to the written mathematical text.

The semantic disjunction between spoken discourse and mathematical symbolism occurs several times in the lesson. For example, the linguistic nominal group with postmodifying clause 'an expression as ten divided by the difference between two tangent values' (clause 545) has a different meaning to the mathematical expression $\frac{10}{(\tan\theta - \tan\alpha)}$ which is a nuclear configuration of embedded mathematical processes and participants. That is, the linguistic participant 'an expression as [[ten divided by the difference between two tangent values]]' is a nominal group with one embedded clause, while the symbolic $\frac{10}{[[(\tan\theta - \tan\alpha)]]}$ consists of two mathematical processes (division and subtraction) with their accompanying participants. In particular, the grammatical metaphor in the embedded clause 'the difference between two tangent values' is not equivalent to symbolic configuration 'tan θ - tanα'. Therefore, the spoken version of the mathematical symbolic expression results in a metaphorical version of the written mathematics.

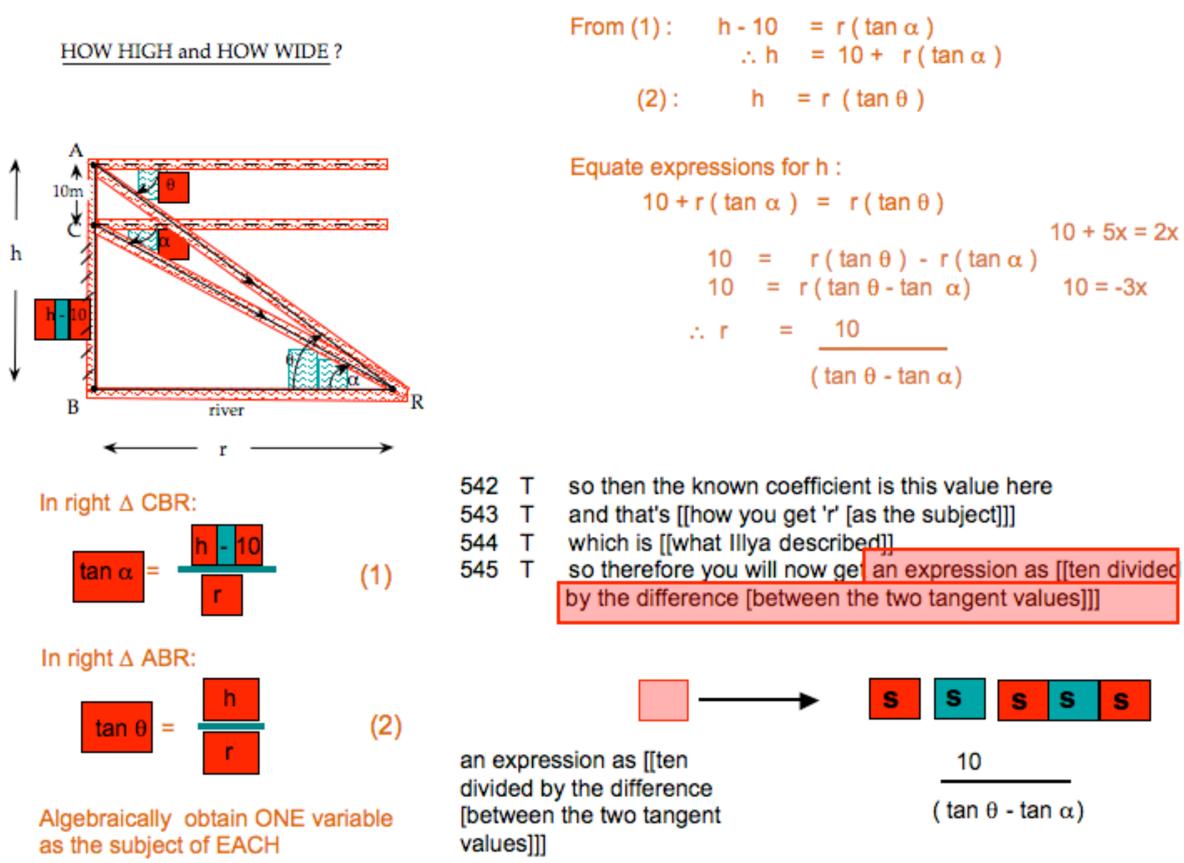


Figure 11.7 Semantic Disjunction between Mathematical Symbolism and Spoken Language

From this discussion, it is evident that teaching and learning mathematics involves the use of multiple semiotic resources which interact inter-semiotically across different modalities in complex ways. Students must move from the material world of spoken discourse and actions to the visual abstract world of mathematics with its specialised grammars, inter-semiotic semantic circuit of linguistic, visual and symbolic elements, and pre-established mathematical results. The unique grammars of language, mathematical images, and mathematical symbolism, the inter-semiotic expansions of meaning which take place as the three resources integrate and mathematical laws, axioms and results come into play as mathematics knowledge is constructed. Furthermore, there are metaphorical expressions of meaning arising from resemioticisation of semiotic choices across different resources and modalities. The semantic space arising from intra-semiosis, inter-semiosis and inter-modal relations in mathematics classrooms are explored below.

The Semantic Hyperspace

The semantic space arising from the meaning potential of one semiotic resource and the modalities through which it materialises are displayed in Figure 11.8(a). System selections, represented as points in Figure 10.8(a), create semantic subspaces which overlap with each other to create a complex space in Figure 10.8(b). However, the semantic spaces arising from one semiotic resource in Figures 10.8(a)-(b) are part of a much larger multidimensional semantic space when multiple semiotic resources are involved. The semantic space arising from the integration of semiotic resources across different modalities is called the semantic hyperspace, following the mathematical definition of hyperspace as a Euclidean space with dimensions higher than three. The semantic hyperspace is displayed in Figure 10.8(c) and the subspaces arising from systemic choices across semiotic resources and modalities in the hyperspace are displayed in Figure 10.8 (d).

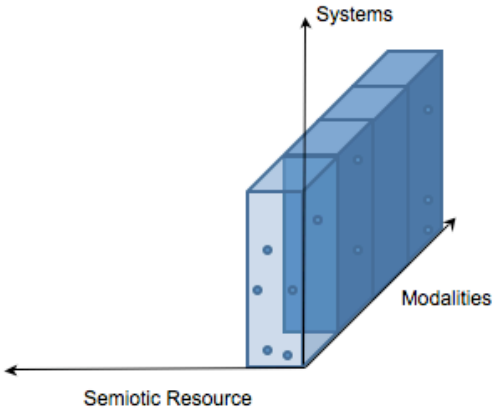


Figure 11.8(a) Semantic Space

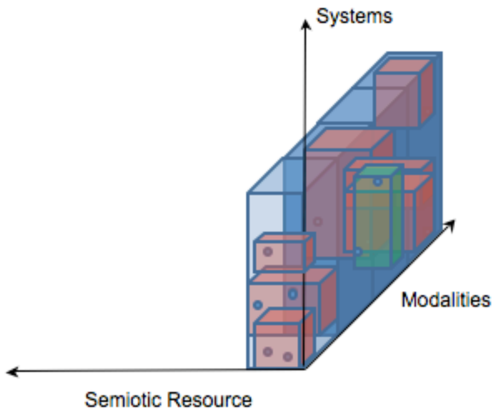


Figure 11.8(b) Subspaces in the Semantic Space

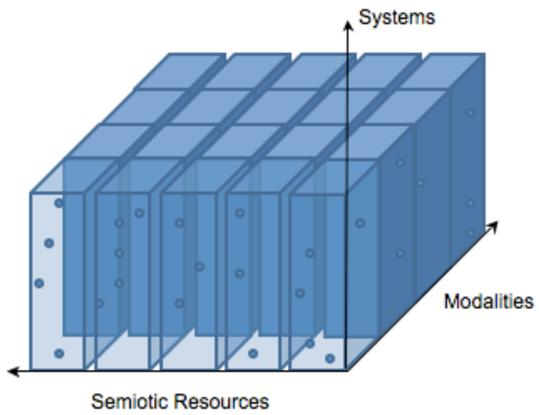


Figure 11.8(c) Semantic Hyperspace

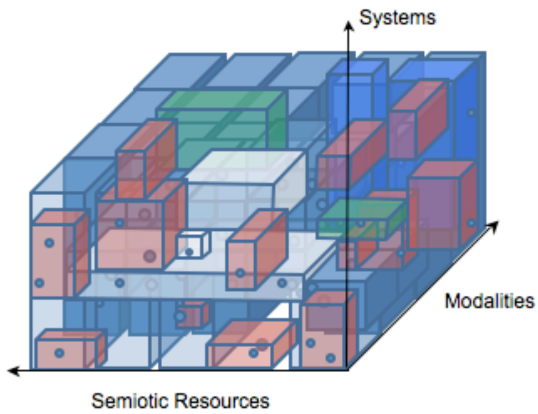


Figure 11.8(d) Subspaces in the Semantic Hyperspace

It is important to realise that the semantic subspaces, spaces and the hyperspace are not three-dimensional spaces as displayed in Figure 11.8(a)-(d). On the contrary, these semantic spaces are complex multidimensional spaces, arising from the sets of systems which function across different ranks, semiotic resources and modalities. For example, system choices operate at the rank of word, word group, clause, clause complex, paragraph and text for language, and at the rank of expressions, clauses and statements for mathematical symbolism (O'Halloran, 2008). Moreover, system choices form different types of semantic structures according to rank, metafunction, semiotic resource and modality, and these semantic structures interact in complex ways. The semantic subspaces, spaces and the hyperspace are not random, however, otherwise it would be impossible for inter-subjective understanding of the semantic spaces which are created. The metafunctional organisation of systemic choices provides one basis for organising and understanding the semantic hyperspace. Furthermore, although not explored in this paper, there are mathematical techniques which can be used to represent the multidimensional semantic space as a manifold where the complexity of the semantic space is expressed and understood in terms of simpler spaces (O'Halloran, Judd, & E., forthcoming).

Experiential and logical meaning are re-contexualised in the hyperspace of mathematics so that expansions of meaning arise through the unique meaning potential of language, image and symbolism, the inter-semiotic expansions of meaning which take place across the three resources, and pre-established mathematical laws, axioms and results. Interpersonal meaning is co-contextualised, keeping a constant backdrop so that the focus is the expansion of experiential and logical meaning in mathematics. Significantly, the focus on experiential and logical meaning is enabled by the sophisticated systems of textual organisation in mathematical diagrams and the symbolism which exploit the spatiality of the page.

Accumulating mathematical knowledge in the hyperspace of the mathematics classroom involves moving from the material world with its semiotic resources which materialise across a range of modalities into the hyperspace of mathematics with its specialised semiotic resources, semantic expansions which take place through inter-semiosis and the pre-established mathematics results. In addition, the hyperspace of the classroom involves semantic disjunctions, particularly when the mathematical content is verbalised. Thus we may come to understand more about the nature of mathematical knowledge and the difficulties students face in learning mathematics. Mathematics involves semantic expansions of the experiential realm in logically deductive structures created through language, image and symbolism that function together to construct meaning in ways which differ from that found in the material world. The students enter into a hyperspace with specialised semiotic resources with visual modality which they need to understand in relation to the semiotic resources and modalities used in the construction of everyday reality.

Conclusion

Mathematics is more than the sum of the meaning potential of the three semiotic resources of language, image and symbolism. The semantic circuit in mathematics means that inter-semiosis gives rise to semantic expansions across different semiotic resources whereby semiotic choices are resemioticised into different functional elements (e.g. a linguistic configuration of process, participant and circumstance is resemioticed as a visual entity). Moreover, the semantic structure of semiotic choices may be downranked or upranked as the semantic circuit unfolds (e.g. visual participant is resemioticised as a configuration of mathematical processes and participants). In addition, mathematical knowledge is constructed using previously established results. The trigonometric problem considered in this chapter is solved precisely in this manner. The teacher moves from linguistic and visual formulation of the problem to the abstract symbolic solution using previously established mathematical results, and throughout each stage of this process, inter-semiotic expansions of meaning take place across semiotic resources and modalities.

However, there exists the potential for semantic disjunction when spoken language functions as the metadiscourse for the written mathematical texts. In some cases, the meaning of the written symbolic mathematics is not be equivalent to the spoken language because symbolic mathematics draws upon spatiality and unique grammatical strategies (e.g. ellipsis of operations, rules of order and brackets) to encode meaning. Quite simply, talking mathematics is not the same as doing mathematics. This means that teachers must engage the students directly with the written mathematics texts, and this is one major reason why the board demonstration is a common activity in mathematics classrooms. This observation brings into question pedagogical practices which indiscriminately advocate material activities and spoken discussions about the mathematics. Certainly these activities form valuable components of the mathematics curriculum but ultimately students must move from the material world into the abstract visual world of mathematics and use the appropriate semiotic resources accordingly. Students must understand and use the unique grammatical resources of mathematical images and symbolism which function differently to language, and they must be able to navigate their way through the semantic circuit of language, image and symbolism to solve mathematical problems. Most importantly, the discipline of mathematics is vertical in nature (Christie & Martin, 2007), so students must have a strong foundational knowledge upon which to build their mathematical expertise. This is best achieved through a structured mathematics curriculum with competent teachers who can stage and specifically guide the building of mathematical knowledge in the classroom.

A multisemiotic and inter-modal approach to mathematics sheds light on the semantic hyperspace of mathematics and the hierarchical knowledge which is required. It also sheds light on the hyperspace of the mathematics classroom to reveal the semantic disjunctions which occur when teachers and students talk about mathematics. In addition, the approach reveals how semantic expansions made possible through inter-semioisis have played an important role in the development of scientific language, particularly grammatical metaphor. The development of mathematical images and the symbolism as semiotic resources must have impacted on scientific writing, particularly given the role language was expected to play in relation to the functions of the other two resources.

In conclusion, we may return to Halliday's (1978: 195-196) explanation that mathematics is not just a question of new words, but also new 'styles of meaning and modes of argument ... and of combining existing elements into new combinations'. This is particularly true when one considers mathematics and mathematics classrooms as semantic hyperspaces in which meaning is negotiated across multiple semiotic resources and modalities.

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